Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. (Mathematics) (2019 Batch) (Sem.–2)

ALGEBRA-I

Subject Code : MSM-101

M.Code : 74720

Time : 3 Hrs. **Max. Marks : 80**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions in each section carrying SIXTEEN marks each.**
3. **Select atleast TWO questions from SECTION - B & C EACH.**

SECTION-A

1. Answer briefly :

(a) Prove that $/^* = / - \{0\}$ is an Abelian group under multiplication.

(b) Show that intersection of two subgroups of a group G is also a subgroup of G.

(c) Define quotient group and give an example.

(d) Determine whether the permutation (1,2,3,4,5)(1,2,3)(4,5) is even or not?

(e) Define a subnormal series of a group and give an example.

(f) State Fundamental theorem on finite Abelian groups.

(g) Is Z an ideal of Q? Justify.

(h) Define a simple ring and give an example.

SECTION-B

2. (a) Show that every subgroup of an Abelian group is normal. [8]

(b) If N and M are two normal subgroups of a group G, show that NM is also normal subgroup of G. [8]

3. (a) Prove that no group of order 108 is simple. [6]

(b) Let G be a group with $O(G) = pq, p, q$ are distinct primes. Show that G is cyclic. [10]

4. (a) Show that a group G is solvable if and only if $G^{(n)} = (e)$ for some non-negative integer n. [10]

(b) Find the derived subgroup of S_3. [6]

SECTION-C

5. (a) State and prove Sylow's third theorem. [10]

(b) Show that a group of order 36 has either 1 or 4 Sylow 3-subgroups. [6]

6. (a) Show that any finite additive Abelian group is internal direct product of its Sylow subgroups. [10]

(b) If I and J arc two ideals of a ring R, then show that $I \cup J$ is an ideal of R if and only if either I [6]

7. (a) State and Prove Fundamental theorem of Ring homomorphism. [10]

(b) Give an example of a maximal ideal of a ring R, which is not prime. [6]

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.